The discussion has already reached a greater length than I anticipated; for, as I have already said, it is not my intention to undertake a systematic re-examination of Pritchard's work. Starting such work now one would do it differently perhaps; we have had ten years' experience in photographic measurement since Pritchard began. But that his work can be dismissed in the cursory manner of Sir David Gill's Introduction I do not believe. It is at least worthy of careful examination. For one thing it is (so far as I know) even yet the only considerable research on stellar parallax by photography actually carried out. Many peeple have talked of doing such work before and since, but as compared with talk, an actual piece of work done is entitled to a measure of respect.

The Normal Equations that arise in the usual Schemes of Observation for Division Errors and their Solutions. By P. H. Cowell, M.A.

Notation.

The circle of 360° is supposed divided into n arcs of $\frac{360°}{n}$ each; and the error of the division $r\frac{360°}{n}+A°$ is denoted by x_r, where r takes all the values 1, 2 . . . n. At the first setting of the circle the $s+1$th division at the $A+(s+1)\frac{360}{n}$th degree is supposed to be under the sth microscope. In some cases s will take all integral values from 1 to n, but in other cases s will only take some of these values. Between the settings the graduated circle is supposed to be turned in such a way that the $s+t$th division at the $A+(s+t)\frac{360}{n}$th degree is under microscope s. The series of observations will be complete when there have been n settings of the circle, but in order to eliminate time changes it is usual to at once repeat the observations in the reverse order. Any uniform change is eliminated either by taking the mean of corresponding readings, that is to say, readings equidistant from the middle of the double series, or preferably by solving each single series separately, and taking the mean of the final results, whose discordance indicates the amplitude of the changes that may have occurred.

It is henceforth assumed that a single series of observations is being dealt with.

The division error x_r is defined as the amount by which the circle appears to read too large in consequence of division error. The sth microscope is supposed subject to an error of position

y_s, causing the reading under the microscope to be too large. At the tth setting the circle will be supposed to have been turned too far by a distance z_t, so that all the readings at the tth setting are on this account z_t larger than was intended.

The Equations.

If $c_{s.t}$ be the reading under the sth microscope at the tth setting, the typical equation is

$$x_r + y_s + z_t = c_{st}$$

where

$$r = s + t$$

and r or t may have any integral value from 1 to n, and s may have certain selected values according to the number and positions of the microscopes. Let m be the number of microscopes; then we have mn equations of the above type to determine $2n+m$ quantities, of which two, say x_n and z_n, are arbitrary. If, therefore, mn is greater than $2n+m-2$, we have more equations than unknown quantities, and theoretically normal equations should be employed. Since, however, the normal solution is the best, that is to say, of maximum accuracy, any solution, where the fundamental equations are combined not very differently, will have an accuracy hardly appreciably less; there is, on the other hand, a risk that the fundamental equations, if combined arbitrarily, may be combined in a way that may give a very erroneous result. As an illustration, if $x=1$ and $x=3$ be two equations of equal weight, the normal solution gives $x=2$; but any other value for x may be obtained by unequal weighting. To return to the system of equations under discussion, if all the quantities except x_r be eliminated in an arbitrary fashion is there any guarantee that the process is not equivalent to assigning a negative weight to some of the equations?

The Normal Equations.

The normal equations formed in the usual manner are:

$$\begin{aligned} mx_r \quad &+ \Sigma y + \Sigma_s z_{r-s} = \Sigma_s c_{s.r-s} \\ \Sigma x \quad &+ ny_s + \Sigma z \quad = \Sigma_t c_{s.t} \\ \Sigma_s x_{s+t} &+ \Sigma y + mz_t \quad = \Sigma_s c_{s.t} \end{aligned}$$

When the suffix of Σ denotes the symbol for all values of which the summation is to be taken, and when the suffix is omitted, the summation includes all quantities expressed by the same letter.

It is to be remarked that there are $2n+m$ equations written above to determine $2n+m$ quantities of which two are arbitrary. As, however, the sum of all the equations in a group is the same

for each of the three groups, there are only $2n+m-2$ independent equations.

As the y's do not occur in the first and third groups except in the single form Σy, it is clear that we may treat the m equations of the second group as determining the m quantities y; we may therefore suppress this group, and we are left with

$$mx_r \quad +\Sigma y+\Sigma_s z_{r-s}=\Sigma_s c_{s\,.\,r-s}$$
$$\Sigma_s x_{s+t}+\Sigma y+mz_t \quad =\Sigma_s c_{s\,.\,t}$$

or $2n-1$ independent equations, to determine n quantities x, n quantities z, and one quantity Σy, or $2n+1$ quantities in all, of which two, x_n and z_n say, are arbitrary.

Particular Cases.

(i.) $n=6$; $m=6$.

The normal equations are

$$6x_r=\Sigma_s c_{s\,.\,r-s}$$
$$6z_t=\Sigma_s c_{s\,.\,t}$$

where the arbitrary constants have been chosen so that

$$\Sigma x=\Sigma z=-\Sigma y.$$

The most convenient form for computation is to enter the 36 readings in a square of six by six, the readings in the same row corresponding to the same setting of the circle, and the readings in the same column to the same division of the circle, and the readings of the same microscope will fall on a line parallel to one of the diagonals.

Let S denote a sum taken vertically, and S′ a sum taken horizontally; then

$$6x_r=S_r \qquad 6z_t=S'_t$$

and as a numerical check S′S=SS′, or the sum taken horizontally of the sums S is equal to the sum taken vertically of the sums S′.

If the weight of a single reading be unity, the weight of a determination of division error is 6.

(ii.) $n=8$; $m=7$.

Let the missing microscope be the last one; the case supposed sometimes occurs when it is inconvenient or impossible to mount an eighth microscope.

The normal equations are

$$7x_r-z_r \quad =\Sigma_s c_{s\,.\,r-s}$$
$$-x_t+7z_t=\Sigma_s c_{s\,.\,t}$$

where the arbitrary constants are chosen so that

$$\Sigma x=\Sigma z=-\Sigma y.$$

Arrange the 56 readings in a square of eight by eight with a diagonal empty to correspond to the missing microscope. As before, readings on the same row correspond to the same setting; readings in the same column to the same division. Let S denote the sum of the seven quantities in a column, S′ the sum of the seven quantities in a row; then

$$7x_r - z_r = S_r$$
$$-x_r + 7z_r = S'_r;$$

then eliminating z_r

$$48x_r = 7S_r + S'_r$$

To finish the computations, multiply the quantities S by 7 in a row beneath them, and add the quantities S′. As a check $S'S = SS'$ as before; also $48\Sigma x = 8S'S$.

If the weight of a single reading be unity, the weight of a determination of division error is $\frac{1152}{175} = 6{\cdot}58$.

(iii.) $n=24$; $m=8$, s taking the values 1, 4, 8, 12, 13, 16, 20, 24. This is a usual way of obtaining the division errors for 15°. It is usually preceded by a determination of every 60°. In fact, by suppressing the readings of the microscopes $s=1$, $s=13$, we arrive at four repetitions of case (i.), so that the division errors of four sets of divisions, each set being a group of six 60° apart, may be determined as in case (i.) each to an arbitrary constant. It is the function of the two microscopes $s=1$, $s=13$ to connect the four arbitrary constants of the four groups.

The normal equations are—

$$8x_r \quad + \Sigma y + \Sigma_s z_{r-s} = \Sigma_s c_{s\,.\,r-s}$$
$$\Sigma_s x_{s+t} + \Sigma y + 8z_t \quad = \Sigma_s c_{s\,.\,t}$$

The eliminations, in order to obtain the values of the quantities X separately, are difficult to perform, and therefore, for the reasons given above, equations for four quantities X_p ($p=1, 2, 3, 4$)

where $$X_p = \Sigma_k x_{4k+p} \; (k=1, 2, 3, 4, 5, 6)$$

will be formed. A similar meaning being attached to the symbols Z_p and $C_{s\,.\,p-s}$ and $C_{s\,.\,p}$, we have, by adding the normal equations in groups of six—

$$8X_p \quad + 6\Sigma y + \Sigma_s Z_{p-s} = \Sigma_s C_{s\,.\,p-s}$$
$$\Sigma_s X_{s+p} + 6\Sigma y + 8Z_p \quad = \Sigma_s C_{s\,.\,p}$$

Now $$\Sigma_s Z_{p-s} = 6Z_p + 2Z_{p-1}$$
$$\Sigma_s X_{s+p} = 6X_p + 2X_{p+1}$$

Again, the $24 \times 8 = 192$ readings of the complete set may be grouped as follows: Four groups of 36 readings each, namely, the readings of the six symmetrical microscopes at the pth, $p+4$th, $p+8$th, $p+12$th, $p+16$th, $p+20$th settings; let the

sum of one group of 36 readings be C_p. Four groups of 12 readings each, namely, the readings of the two unsymmetrical microscopes at the same settings; let the sums be denoted by C'_p ($p=1, 2, 3, 4$), then it is not difficult to see that

$$\Sigma_s C_{s.p-s} = C_p + C'_{p-1}$$
$$\Sigma_s C_{s.p} \;\; = C_p + C'_p$$

and the eight normal equations become:

$$8X_p + 6\Sigma y + 6Z_p + 2Z_{p-1} = C_p + C'_{p-1}$$
$$6X_p + 2X_{p+1} + 6\Sigma y + 8Z_p = C_p + C'_p$$

or eliminating the Z's:

$$12(2X_p - X_{p-1} - X_{p+1}) = 8C_p + 8C'_{p-1} - 6C_p - 6C'_p - 2C_{p-1} - 2C'_{p-1}$$
$$= 2C_p - 2C_{p-1} + 6C'_{p-1} - 6C'_p$$

Solving this difference equation

$$6(X_p - X_{p-1}) = 3C'_{p-1} - C_{p-1}$$

plus an arbitrary constant, which is determined by the condition

$$\Sigma(X_p - X_{p-1}) = 0.$$

Hence the following precepts for computing are obtained:—

Form the four quantities $D_p = 3C'_p - C_p$ (to do this subtract the sum of the readings of the six symmetrical microscopes for each setting of the circle from three times the sum of the two unsymmetrical readings, and then add the 24 quantities thus obtained in four groups of six); then

$$6X_p = \tfrac{3}{8}(D_{p-1} - D_p) + \tfrac{1}{8}(D_{p-2} - D_{p-1})$$

and if the weight of a single observation be unity, the weight of the determination of $\frac{X_p}{6}$ is $\frac{144}{5} = 28{\cdot}8$.

On a modified form of Revolving Occulter for adapting the exposure of the Sun's Corona to its actinic intensity at all distances from the Moon's limb. By Professor David P. Todd, Director of Amherst College Observatory.

(*Communicated by the Secretaries.*)

Fifty years have now elapsed since the Sun's corona was first daguerreotyped. A heliometer with uncorrected objective was used, and it is not certain that the clockwork was sufficiently good to have prevented the blurring inseparable from the long exposure required by the relatively insensitive silver iodised surface. Still, the daguerreotype of the corona of 1851 compares very favourably with many photographs of the most recent